Information Filter and Kalman Filter Comparison: Selection of the Faster Filter

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***Abstract*— A comparison study is presented between the discrete time Kalman filter and the Information filter, which are equivalent with respect to their behavior, since they produce the same estimations. The computational requirements of the Kalman and Information filters are determined and a method is proposed to a-priori (before the filters' implementation) decide which filter is the faster one.**

***Keywords*— Kalman Filter; Information Filter; Time Varying System; Time Invariant System; Estimation**

I. Introduction

Estimation plays an important role in many fields of science: applications to aerospace industry, chemical process, communication systems design, control, civil engineering, filtering noise from 2-dimensional images, pollution prediction and power systems are mentioned in [1]. The estimation problem arises in linear estimation and is associated with discrete time systems described by the following state space equations:

$$\begin{aligned} x_{k+1} &= F_k x_k + w_k \\ z_k &= H_k x_k + v_k \end{aligned} \tag{1}$$

for $k \geq 0$, where x_k is the $n \times 1$ state vector, z_k is the $m \times 1$ measurement vector, F_k is the $n \times n$ transition matrix, H_k is the $m \times n$ output matrix, w_k is the $n \times 1$ state noise vector and v_k is the $m \times 1$ measurement noise vector at time k . Also, $\{w_k\}$ and $\{v_k\}$ are independent Gaussian zero-mean white and uncorrelated random processes, Q_k and R_k are the $n \times n$ plant and $m \times m$ measurement noise covariance matrices respectively and x_0 is a Gaussian random process with mean $\overline{x}_0$ and covariance P_0 .

The filtering problem is to produce the optimal estimation $x_{k/k}$ of the state vector at time k given the measurements set $Z_k = \{z_0, z_1, z_2, ..., z_k\}$ up till time k .

The $n \times 1$ estimation vector is $x_{k/k} = E[x_k / Z_k]$ and the $n \times n$ estimation error covariance is $P_{k/k} = E[(x_k - x_{k/k})(x_k - x_{k/k})^T / Z_k]$.

The $n \times 1$ prediction vector is $x_{k+1/k} = E[x_{k+1} / Z_k]$ and the $n \times n$ prediction error covariance is $P_{k+1/k} = E[(x_{k+1} - x_{k+1/k})(x_{k+1} - x_{k+1/k})^T / Z_k]$.

The discrete time Kalman filter [1] and Information filter [1], [6], [8] are well known algorithms that solve the filtering problem. Both filters have been used in various applications: Kalman filter applications are referred in [1], [2], [4], [10], while Information filter applications are mentioned in [4], [5], [7], [9], [11].

In this paper, a comparison study for the Kalman and Information filters is presented. The paper is organized as follows: The Kalman filter and the Information filter are presented in sections 2 and 3, respectively. The computational requirements of the Kalman and Information filters are determined in section 4. A method is proposed to decide which filter is the faster one in section 5. Finally, section 6 summarizes the conclusions.

II. Kalman Filter

For *time varying systems*, the **Time Varying Kalman Filter** (**TVKF**) [1] is summarized in the following:

$$\begin{aligned} K_k &= P_{k/k-1} H_k^T [H_k P_{k/k-1} H_k^T + R_k]^{-1} \\ x_{k/k} &= [I - K_k H_k] x_{k/k-1} + K_k z_k \\ P_{k/k} &= [I - K_k H_k] P_{k/k-1} \\ x_{k+1/k} &= F_k x_{k/k} \\ P_{k+1/k} &= Q_k + F_k P_{k/k} F_k^T \end{aligned} \tag{2}$$

for $k = 0,1,...$ and with initial conditions $x_{0/-1} = \overline{x}_0$ and $P_{0/-1} = P_0$.

The existence of the inverse matrices that appear in Kalman filter equations is guaranteed in the case where the measurements noise covariances R_k are positive definite, denoted by $R_k > 0$; this happens in the case where no measurement is exact.

For *time invariant systems* where the transition matrix $F = F_k$, the output matrix $H = H_k$, as well as the plant and measurement noise covariance matrices $Q = Q_k$ and $R = R_k$ are constant matrices, the **Time Invariant Kalman Filter** (**TIKF**) is derived.

III. Information Filter

As described in [6], [8], the Information filter uses the Information matrix, which is the inverse $S_{k/k}$ of the covariance matrix $P_{k/k}$, and the Information state vector $y_{k/k}$ which is connected to the estimation vector $x_{k/k}$ through the Information matrix. In fact, the Information filter uses the definitions:

$$
\begin{aligned}
y_{k/k} &= P_{k/k}^{-1} x_{k/k} \\
S_{k/k} &= P_{k/k}^{-1} \\
y_{k/k-1} &= P_{k/k-1}^{-1} x_{k/k-1} \\
S_{k/k-1} &= P_{k/k-1}^{-1}
\end{aligned} \tag{3}
$$

The Information filter is derived using the Kalman filter equations and the Matrix Inversion Lemma [1]: let the $n \times n$ identity matrix I, the $n \times n$ matrix P, the $m \times n$ matrix H and the $m \times m$ matrix R; then the following equatily holds under the assumption that the inverses exist:

$$
\begin{aligned}
[I + PH^T R^{-1} H]^{-1} P &= [P^{-1} + H^T R^{-1} H]^{-1} \\
&= P - PH^T [HPH^T + R]^{-1} HP
\end{aligned} \tag{4}
$$

For *time varying systems*, the **Time Varying Information Filter** (**TVIF**) [1], [6], [8] is summarized in the following:

$$
\begin{aligned}
y_{k/k} &= y_{k/k-1} + H_k^T R_k^{-1} z_k \\
S_{k/k} &= S_{k/k-1} + H_k^T R_k^{-1} H_k \\
P_{k/k} &= S_{k/k}^{-1} \\
x_{k/k} &= S_{k/k}^{-1} y_{k/k} \\
K_k &= S_{k/k}^{-1} H_k^T R_k^{-1} \\
P_{k+1/k} &= Q_k + F_k S_{k/k}^{-1} F_k^T \\
S_{k+1/k} &= P_{k+1/k}^{-1} \\
y_{k+1/k} &= S_{k+1/k} F_k S_{k/k}^{-1} y_{k/k} \\
x_{k+1/k} &= S_{k+1/k+1}^{-1} y_{k+1/k}
\end{aligned} \tag{5}
$$

for $k = 0,1,...$ and with initial conditions $y_{0/-1} = P_{0/-1}^{-1} \bar{x}_0 = P_0^{-1} \bar{x}_0$ and $S_{0/-1} = P_{0/-1}^{-1} = P_0^{-1}$.

The existence of the inverse matrices that appear Information filter equations is guaranteed in the case where the measurements noise covariances R_k are positive definite, i.e. $R_k > 0$; this happens in the case where no measurement is exact. Also, the initial condition P_0 has to be nonsingular.

For *time invariant systems* where the transition matrix $F = F_k$, the output matrix $H = H_k$, as well as the plant and measurement noise covariance matrices $Q = Q_k$ and $R = R_k$ are constant matrices, the **Time Invariant Information Filter** (**TIIF**) is derived. Note that the matrices R^{-1}, $H^T R^{-1}$, $H^T R^{-1} H$ are calculated off-line.

The Information filter equations are derived by the Kalman filter equations. Thus the Information filter equations are algebraically equivalent to the Kalman filter equations [1] and the Kalman and Information filters calculate theoretically the same estimates as well as the same estimation error covariances. This means that the two filters are theoretically equivalent with respect to their performance.

IV. COMPUTATIONAL REQUIREMENTS

The Kalman and Information filters calculate the same estimates. Then, it is reasonable to assume that both the Kalman filter and the Information filter compute the estimate value $x_{k/k}$ of the state vector x_k executing the same number of iterations. Thus, in order to compare the algorithms with respect to their computational time, we have to compare their per step (iteration) calculation burden (CB) required for the on-line calculations; the calculation burden of the off-line calculations (initialization process for time invariant and steady state filters) is not taken into account.

Scalar operations are involved in matrix manipulation operations, which are needed for the implementation of the filtering algorithms. Table I summarizes the calculation burden of needed matrix operations. Note that the identity matrix is denoted by I and a symmetric matrix by S. The details are given in [3]. The per step (iteration) calculation burdens of the Kalman and Information filters are analytically calculated in the Appendix and summarized in Table II.

TABLE I
CALCULATION BURDEN OF MATRIX OPERATIONS

Matrix Operation	Matrix Dimensions	Calculation Burden
$A + B = C$	$(n \times m) + (n \times m)$	nm
$A + B = S$	$(n \times n) + (n \times n)$	$\frac{1}{2}n^2 + \frac{1}{2}n$
$I + A = B$	$(n \times n) + (n \times n)$	n
$A \cdot B = C$	$(n \times m) \cdot (m \times \ell)$	$2nm\ell - n\ell$
$A \cdot B = S$	$(n \times m) \cdot (m \times n)$	$n^2 m + nm - \frac{1}{2}n^2 - \frac{1}{2}n$
$A^{-1} = B$	$(n \times n)$	$\frac{1}{6}(16n^3 - 3n^2 - n)$

TABLE II
PER STEP CALCULATION BURDEN OF FILTERS

System	Filter	Calculation Burden
Time Varying	Kalman Filter	$CB_{TVKF} = 4n^3 + \frac{7}{2}n^2 - \frac{3}{2}n + 4n^2 m + nm + 3nm^2 + \frac{1}{6}(16m^3 - 3m^2 - m)$
	Information Filter	$CB_{TVIF} = \frac{1}{3}(25n^3 + 21n^2 - 13n) + 3n^2 m + nm + 2nm^2 + \frac{1}{6}(16m^3 - 3m^2 - m)$
Time Invariant	Kalman Filter	$CB_{TIKF} = 4n^3 + \frac{7}{2}n^2 - \frac{3}{2}n + 4n^2 m + nm + 3nm^2 + \frac{1}{6}(16m^3 - 3m^2 - m)$
	Information Filter	$CB_{TIIF} = \frac{1}{6}(50n^3 + 45n^2 - 23n) + 2n^2 m + nm$

V. SELECTION OF THE FASTER FILTER

In the following, a method is proposed to select the faster filter. From Table II, it is clear that the algorithms' calculation burdens depend on the state vector dimension n and the measurement vector dimension m. Thus, the selection of the faster implementation depends on the relationship between n and m.

For *time varying systems*, the difference between the calculation burden required for the time invariant Kalman filter implementation and the calculation burden required for the time invariant Information filter implementation is:

$$
\begin{aligned}
q_{TV} &= CB_{TVKF} - CB_{TVIF} \\
&= \frac{1}{6} n (6m^2 + 6nm - 26n^2 - 21n + 17)
\end{aligned} \tag{6}
$$

Consider the quadratic function of m :

$$\hat{q}_{TV}(m) = 6m^2 + 6nm - 26n^2 - 21n + 17$$

Then the equation $\hat{q}_{TV}(m) = 0$ has a positive real root $\rho_{TV}(n) = \dfrac{-6n + \sqrt{660n^2 + 504n - 408}}{12}$, since the discriminant $\Delta = 660n^2 + 504n - 408$ is a positive number and the product of its real roots is equal to $\frac{1}{6}(-26n^2 - 21n + 17) < 0$, for every $n \geq 1$.

Hence, if $m > \rho_{TV}(n)$, then it is implied that $6m^2 + 6nm - 26n^2 - 21n + 17 > 0$. Thus, for time varying systems, when $1 \leq m \leq \rho_{TV}(n)$, the faster filter is the Kalman filter while when $m > \rho_{TV}(n)$, the faster filter is the Information filter.

For time varying systems, the areas where the time invariant Kalman or Information filter implementation is faster, for various values of the model order ($n = 1,...,100$ and $m = 1,...,100$) are shown in Fig. 1.

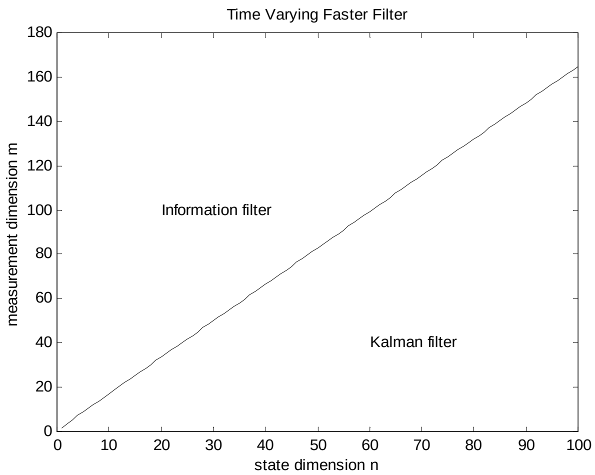

Fig. 1 The faster time varying filter

From Fig. 1, it is easy to conclude the following Rule of Thumb for time varying systems: the time varying Information filter is faster than the time varying Kalman filter, when the following relation holds:

$$m > 1.65n \tag{7}$$

For *time invariant systems*, the difference between the calculation burden required for the time invariant Kalman filter implementation and the calculation burden required for the time invariant Information filter implementation is:

$$\begin{aligned} q_{TI} &= CB_{TIKF} - CB_{TIIF} \\ &= \tfrac{8}{3}\left(m^3 + \frac{18n-3}{16}m^2 + \frac{12n^2-1}{16}m - \frac{26n^3+24n^2-14n}{16}\right) \end{aligned} \tag{8}$$

Consider the cubic function of m :

$$\begin{aligned} \hat{q}_{TI}(m) &= m^3 + \frac{18n-3}{16}m^2 + \frac{12n^2-1}{16}m - \frac{26n^3+24n^2-14n}{16} \\ &= m^3 + a_2m^2 + a_1m + a_0 \end{aligned}$$

Then by [12, p. 362-365] the solution of the cubic equation $\hat{q}_{TI}(m) = 0$ is related to the intermediate variables

$$\begin{aligned} \hat{Q} &= \tfrac{1}{9}(3a_1 - a_2^2) \\ &= \tfrac{1}{768}(84n^2 + 36n - 19) \end{aligned}$$

$$\begin{aligned} \hat{R} &= \tfrac{1}{54}(9a_1a_2 - 27a_0 - 2a_2^3) \\ &= \tfrac{1}{4096}(3688n^3 + 3084n^2 - 1858n + 9) \end{aligned}$$

and the discriminant

$$\begin{aligned} D &= \hat{Q}^3 + \hat{R}^2 \\ &= \tfrac{1663}{2048}n^6 + \tfrac{11121}{8192}n^5 - \tfrac{2049}{8192}n^4 - \tfrac{22275}{32768}n^3 \\ &+ \tfrac{41113}{196608}n^2 - \tfrac{125}{65536}n - \tfrac{73}{7077888} \end{aligned}$$

Since $\hat{Q} > 0$ it is obvious that $D > 0$; thus there exist two complex conjugates roots $\rho_1(n)$, $\rho_2(n)$ and one real root $\rho_{TI}(n)$, [12, p.364].

Moreover, the real root $\rho_{TI}(n)$ is a positive number, since

$$\rho_1(n)\rho_2(n) = \rho_1(n)\overline{\rho_1(n)} = |\rho_1(n)|^2$$

and

$$\rho_1(n)\rho_2(n)\rho_{TI}(n) = \frac{26n^3 + 24n^2 - 14n}{16} > 0,$$

for every $n \geq 1$.

Hence, if $m > \rho_{TI}(n)$, it is implied that $\hat{q}_{TI}(m) > 0$. Thus, for time invariant systems, when $1 \leq m \leq \rho_{TI}(n)$, the faster filter is the Kalman filter, while when $m > \rho_{TI}(n)$, the faster filter is the Information filter.

For time invariant systems, the areas where the time invariant Kalman or Information filter implementation is faster, for various values of the model order ($n = 1,...,100$ and $m = 1,...,100$) are shown in Fig. 2.

From Fig. 2, it is easy to conclude the following Rule of Thumb for time invariant systems: the time varying Information filter is faster than the time varying Kalman filter, when the following relation holds:

$$m > 0.75n \tag{9}$$

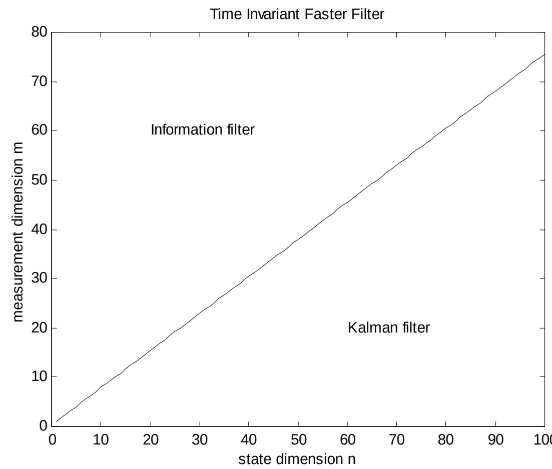

Fig. 2 The faster time invariant filter

This result is confirmed through the following examples taken from [3].

Example 1. An AR type model is considered in this example, where $n=3$ and $m=1$. The speedup from Information filter to Kalman filter is equal to:

$$speedup_{TIIF/TIKF}=\frac{CB_{TIIF}}{CB_{TIKF}}=1.6324$$

Thus, the time invariant Kalman filter is 1.6 times faster than the time invariant Information filter.

Example 2. A typical multisensor problem (seismic signal processing) is considered in this example, where $n=4$ and $m=1000$. The speedup from Kalman filter to Information filter is equal to:

$$speedup_{TIKF/TIIF}=\frac{CB_{TIKF}}{CB_{TIIF}}=7.309991828\cdot 10^{4}$$

Thus, the time invariant Information filter is 73100 times faster than the time invariant Kalman filter.

VI. CONCLUSIONS

The Kalman filter and the Information filter are equivalent with respect to their behavior, since they produce the same estimations and estimation error covariances. A comparison study between the discrete time Kalman and the Information filter was presented. The computational requirements of the Kalman and Information filters were determined and it was pointed out that they depend on the state vector dimension n and the measurement vector dimension m. A method is proposed to a-priori (before the filters' implementation) decide which filter is the faster one: the Information filter is faster than the Kalman filter when state vector dimension n is grater enough than and the measurement vector dimension m; in fact the Information filter is faster than the Kalman filter when $m>1.65n$ for time varying systems and when $m>0.75n$ for time invariant systems.

REFERENCES

[1] B. D. O. Anderson and J. B. Moore, *Optimal Filtering*, Dover Publications, New York, 2005.

[2] U. Arumugam, N. M. Nor and M. F. Abdullah, "A Brief Review on Advances of Harmonic State Estimation Techniques in Power Systems", *International Journal of Information and Electronics Engineering*, vol. 1, no. 3, pp. 217-222, 2011.

[3] N. Assimakis and M. Adam, "Discrete time Kalman and Lainiotis filters comparison", *Int. Journal of Mathematical Analysis (IJMA)*, vol. 1, no. 13, pp. 635-659, 2007.

[4] C. Cadena, J. Neira, "SLAM in O(log n) with the Combined Kalman-Information Filter", *Robotics and Autonomous Systems*, vol. 58, pp. 1207-1219, 2010.

[5] D. W. Casbeer and R. Beard, "Distributed Information Filtering using Consensus Filters", *American Control Conference*, pp. 1882-1887, 2009.

[6] M. E. Khan, Matrix Inversion Lemma and Information Filter, [Online]. Available: http://www.cs.ubc.ca/~emtiyaz/Writings/MILandIF.pdf

[7] Deok-Jin Lee, "Nonlinear Estimation and Multiple Sensor Fusion Using Unscented Information Filtering", *IEEE Signal Processing Letters*, vol. 15, pp. 861-864, 2008.

[8] A. G. O. Mutambara, *Decentralized Estimation and Control for Multisensor Systems*, Boca Ratón, EUA : CRC Press, 1998.

[9] B. Noack, D. Lyons, M. Nagel and U. D. Hanebeck, "Nonlinear information filtering for distributed multisensor data fusion", *American Control Conference (ACC)*, pp. 4846-4852, 2011.

[10] B. Ristic, S. Arulampalam and N. Gordon, *Beyond the Kalman Filter*, Artech House, Boston, London, 2004.

[11] M. H. Terra, J. Y. Ishihara and A. C. J. Padoan, "Information Filtering and Array Algorithms for Descriptor Systems Subject to Parameter Uncertainties", *IEEE Transactions on Signal Processing*, vol. 55, no. 1, 2007.

[12] E. W. Weisstein, *CRC Concise Encyclopedia of Mathematics*, CRC Press, New York, 1998.

APPENDIX

Time Varying Kalman Filter (TVKF)		
Matrix Operation	**Matrix Dimensions**	**Calculation Burden**
$H_k P_{k/k-1}$	$(m\times n)\cdot(n\times n)$	$2n^2m-nm$
$H_k P_{k/k-1} H_k^T$	$(m\times n)\cdot(n\times m)$	nm^2+nm $-\frac{1}{2}m^2-\frac{1}{2}m$
$O_k=H_k P_{k/k-1} H_k^T$ $+R_k$	$(m\times m)+(m\times m)$	$\frac{1}{2}m^2+\frac{1}{2}m$
O_k^{-1}	$(m\times m)$	$\frac{1}{6}(16m^3-3m^2-m)$
$K_k=P_{k/k-1}H_k^T O_k^{-1}$	$(n\times m)\cdot(m\times m)$	$2nm^2-nm$
$K_k H_k$	$(n\times m)\cdot(m\times n)$	$2n^2m-n^2$
$I-K_k H_k$	$(n\times n)+(n\times n)$	n
$[I-K_k H_k]x_{k/k-1}$	$(n\times n)\cdot(n\times 1)$	$2n^2-n$
$K_k z_k$	$(n\times m)\cdot(m\times 1)$	$2nm-n$
$x_{k/k}=K_k z_k+$ $[I-K_k H_k]x_{k/k-1}$	$(n\times 1)+(n\times 1)$	n
$P_{k/k}=$ $[I-K_k H_k]P_{k/k-1}$	$(n\times n)\cdot(n\times n)$	$n^3+\frac{1}{2}n^2-\frac{1}{2}n$
$x_{k+1/k}=F_k x_{k/k}$	$(n\times n)\cdot(n\times 1)$	$2n^2-n$
$F_k P_{k/k}$	$(n\times n)\cdot(n\times n)$	$2n^3-n^2$
$F_k P_{k/k} F_k^T$	$(n\times n)\cdot(n\times n)$	$n^3+\frac{1}{2}n^2-\frac{1}{2}n$
$P_{k+1/k}=Q_k$ $+F_k P_{k/k} F_k^T$	$(n\times n)+(n\times n)$	$\frac{1}{2}n^2+\frac{1}{2}n$
$CB_{TVKF}=4n^3+\frac{7}{2}n^2-\frac{3}{2}n+4n^2m+nm+3nm^2+\frac{1}{6}(16m^3-3m^2-m)$		

Time Invariant Kalman Filter (TIKF)		
Matrix Operation	**Matrix Dimensions**	**Calculation Burden**
$HP_{k/k-1}$	$(m\times n)\cdot(n\times n)$	$2n^2m-nm$
$HP_{k/k-1}H^T$	$(m\times n)\cdot(n\times m)$	nm^2+nm $-\frac{1}{2}m^2-\frac{1}{2}m$
$O_k=HP_{k/k-1}H^T$ $+R$	$(m\times m)+(m\times m)$	$\frac{1}{2}m^2+\frac{1}{2}m$
O_k^{-1}	$(m\times m)$	$\frac{1}{6}(16m^3-3m^2-m)$
$K_k=P_{k/k-1}H^T O_k^{-1}$	$(n\times m)\cdot(m\times m)$	$2nm^2-nm$
$K_k H$	$(n\times m)\cdot(m\times n)$	$2n^2m-n^2$
$I-K_k H$	$(n\times n)+(n\times n)$	n
$[I-K_k H]x_{k/k-1}$	$(n\times n)\cdot(n\times 1)$	$2n^2-n$
$K_k z_k$	$(n\times m)\cdot(m\times 1)$	$2nm-n$
$x_{k/k}=K_k z_k+$ $[I-K_k H]x_{k/k-1}$	$(n\times 1)+(n\times 1)$	n
$P_{k/k}=$ $[I-K_k H]P_{k/k-1}$	$(n\times n)\cdot(n\times n)$	$n^3+\frac{1}{2}n^2-\frac{1}{2}n$
$x_{k+1/k}=Fx_{k/k}$	$(n\times n)\cdot(n\times 1)$	$2n^2-n$
$FP_{k/k}$	$(n\times n)\cdot(n\times n)$	$2n^3-n^2$

$FP_{k/k}F^T$	$(n \times n) \cdot (n \times n)$	$n^3 + \frac{1}{2}n^2 - \frac{1}{2}n$
$P_{k+1/k} = Q + FP_{k/k}F^T$	$(n \times n) + (n \times n)$	$\frac{1}{2}n^2 + \frac{1}{2}n$
$CB_{TIKF} = 4n^3 + \frac{7}{2}n^2 - \frac{3}{2}n + 4n^2m + nm + 3nm^2 + \frac{1}{6}(16m^3 - 3m^2 - m)$		

Time Varying Information Filter (TVIF)		
Matrix Operation	**Matrix Dimensions**	**Calculation Burden**
R_k^{-1}	$(m \times m)$	$\frac{1}{6}(16m^3 - 3m^2 - m)$
$H_k^T R_k^{-1}$	$(n \times m) \cdot (m \times m)$	$2nm^2 - nm$
$H_k^T R_k^{-1} z_k$	$(n \times m) \cdot (m \times 1)$	$2nm - n$
$y_{k/k} = y_{k/k-1} + H_k^T R_k^{-1} z_k$	$(n \times 1) + (n \times 1)$	n
$H_k^T R_k^{-1} H_k$	$(n \times m) \cdot (m \times n)$	$n^2m + nm - \frac{1}{2}n^2 - \frac{1}{2}n$
$S_{k/k} = S_{k/k-1} + H_k^T R_k^{-1} H_k$	$(n \times n) + (n \times n)$	$\frac{1}{2}n^2 + \frac{1}{2}n$
$P_{k/k} = S_{k/k}^{-1}$	$(n \times n)$	$\frac{1}{6}(16n^3 - 3n^2 - n)$
$x_{k/k} = S_{k/k}^{-1} y_{k/k}$	$(n \times n) \cdot (n \times 1)$	$2n^2 - n$
$K_k = S_{k/k}^{-1} H_k^T R_k^{-1}$	$(n \times n) \cdot (n \times m)$	$2n^2m - nm$
$F_k S_{k/k}^{-1}$	$(n \times n) \cdot (n \times n)$	$2n^3 - n^2$
$F_k S_{k/k}^{-1} F_k^T$	$(n \times n) \cdot (n \times n)$	$n^3 + \frac{1}{2}n^2 - \frac{1}{2}n$
$P_{k+1/k} = Q_k + F_k S_{k/k}^{-1} F_k^T$	$(n \times n) + (n \times n)$	$\frac{1}{2}n^2 + \frac{1}{2}n$
$S_{k+1/k} = P_{k+1/k}^{-1}$	$(n \times n)$	$\frac{1}{6}(16n^3 - 3n^2 - n)$
$F_k S_{k/k}^{-1} y_{k/k}$	$(n \times n) \cdot (n \times 1)$	$2n^2 - n$
$y_{k+1/k} = S_{k+1/k} F_k S_{k/k}^{-1} y_{k/k}$	$(n \times n) \cdot (n \times 1)$	$2n^2 - n$
$x_{k+1/k} = P_{k+1/k} y_{k+1/k}$	$(n \times n) \cdot (n \times 1)$	$2n^2 - n$
$CB_{TVIF} = \frac{1}{3}(25n^3 + 21n^2 - 13n) + 3n^2m + nm + 2nm^2 + \frac{1}{6}(16m^3 - 3m^2 - m)$		

Time Invariant Information Filter (TIIF)		
Matrix Operation	**Matrix Dimensions**	**Calculation Burden**
$H^T R^{-1} z_k$	$(n \times m) \cdot (m \times 1)$	$2nm - n$
$y_{k/k} = y_{k/k-1} + H^T R^{-1} z_k$	$(n \times 1) + (n \times 1)$	n
$S_{k/k} = S_{k/k-1} + H^T R^{-1} H$	$(n \times n) + (n \times n)$	$\frac{1}{2}n^2 + \frac{1}{2}n$
$P_{k/k} = S_{k/k}^{-1}$	$(n \times n)$	$\frac{1}{6}(16n^3 - 3n^2 - n)$
$x_{k/k} = S_{k/k}^{-1} y_{k/k}$	$(n \times n) \cdot (n \times 1)$	$2n^2 - n$
$K_k = S_{k/k}^{-1} H^T R^{-1}$	$(n \times n) \cdot (n \times m)$	$2n^2m - nm$
$FS_{k/k}^{-1}$	$(n \times n) \cdot (n \times n)$	$2n^3 - n^2$
$FS_{k/k}^{-1} F^T$	$(n \times n) \cdot (n \times n)$	$n^3 + \frac{1}{2}n^2 - \frac{1}{2}n$
$P_{k+1/k} = Q + FS_{k/k}^{-1} F^T$	$(n \times n) + (n \times n)$	$\frac{1}{2}n^2 + \frac{1}{2}n$
$S_{k+1/k} = P_{k+1/k}^{-1}$	$(n \times n)$	$\frac{1}{6}(16n^3 - 3n^2 - n)$
$FS_{k/k}^{-1} y_{k/k}$	$(n \times n) \cdot (n \times 1)$	$2n^2 - n$
$y_{k+1/k} = S_{k+1/k} FS_{k/k}^{-1} y_{k/k}$	$(n \times n) \cdot (n \times 1)$	$2n^2 - n$
$x_{k+1/k} = P_{k+1/k} y_{k+1/k}$	$(n \times n) \cdot (n \times 1)$	$2n^2 - n$
$CB_{TIIF} = \frac{1}{6}(50n^3 + 45n^2 - 23n) + 2n^2m + nm$		